BTG Winter School ’10

Physical Sciences Session 1 - Mechanics
(Momentum & Impulse, Relative Velocity, Vertical Projectile Motion)

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Force, Momentum & Impulse

Force, Momentum & Impulse

Grade 11 Recap

Newton's First Law Every object will remain at rest or in uniform motion in a straight line unless it is made to change its state by the action of an unbalanced force.

Newton's Second Law The resultant force acting on a body will cause the body to accelerate in the direction of the resultant force The acceleration of the body is directly proportional to the magnitude of the resultant force and inversely proportional to the mass of the object.

Newton's Third Law If body A exerts a force on body B then body B will exert an equal but opposite force on body A.

Newton's Law of Universal Gravitation Every body in the universe exerts a force on every other body. The force is directly proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between them.
Equilibrium Objects at rest or moving with constant velocity are in equilibrium and have a zero resultant force.

Force, Momentum & Impulse

Grade 11 Recap

Equilibrant The equilibrant of any number of forces is the single force required to produce equilibrium.

Triangle Law for Forces in Equilibrium Three forces in equilibrium can be represented in mag- nitude and direction by the three sides of a triangle taken in order.

Momentum The momentum of an object is defined as its mass multiplied by its velocity.

Momentum of a System The total momentum of a system is the sum of the momenta of each of the objects in the system.

Force, Momentum & Impulse

Grade 11 Recap

Principle of Conservation of Linear Momentum: 'The total linear momentum of an isolated system is constant' or 'In an isolated system the total momentum before a collision (or explosion) is equal to the total momentum after the collision (or explosion)'.

Law of Momentum: The applied resultant force acting on an object is equal to the rate of change of the object's momentum and this force is in the direction of the change in momentum.

Force, Momentum & Impulse

New stuff you need to know for finals

Definition: Conservation of Linear Momentum

The total linear momentum of an isolated system is constant. An isolated system has no forces acting on it from the outside.

Force, Momentum & Impulse

Conservation of Linear Momentum

We will consider these in different cases, i.e. 1,2,3 etc.

The total momentum of a system is the total momenta of each of the objects in the system. The total momentum is *always conserved in an isolated system i.e.* there are no external forces acting on the system.

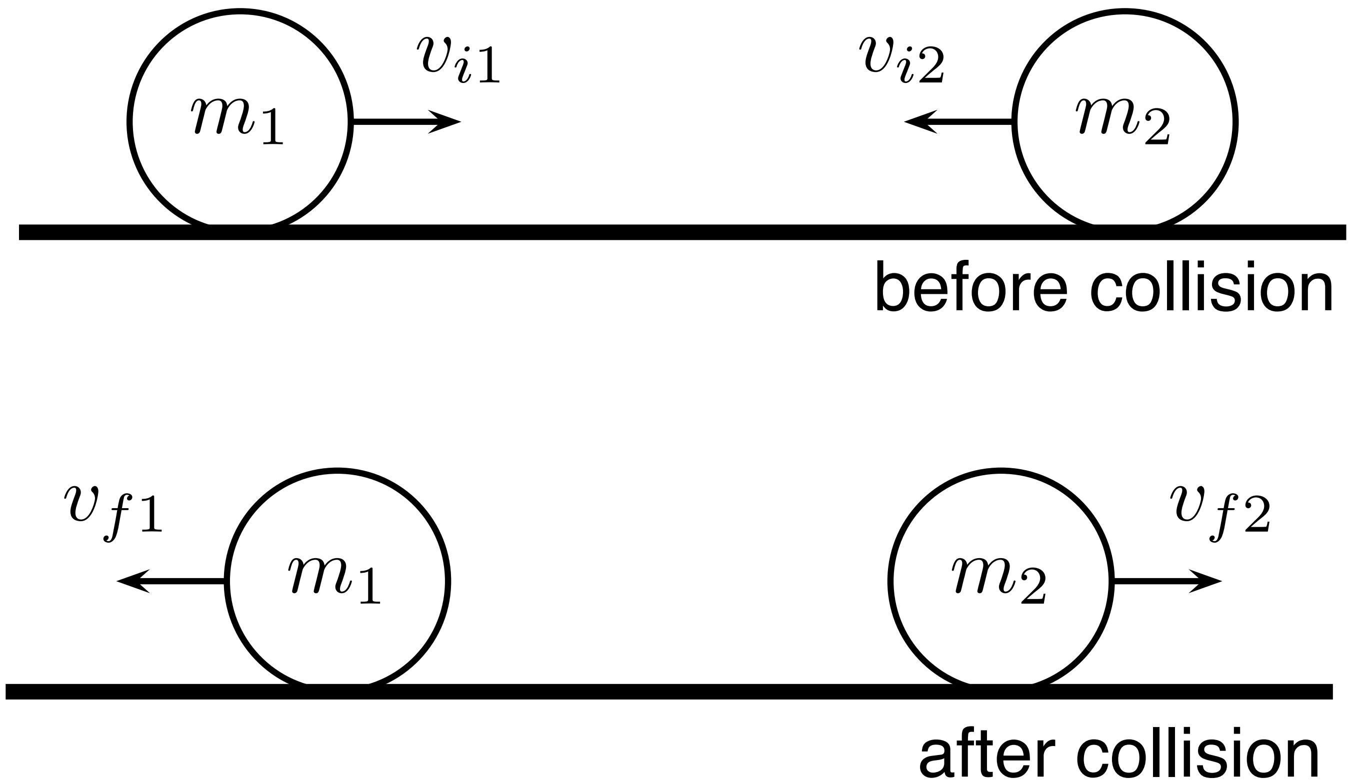
$$p=mv$$

see board explanation for intro to the definition of momentum

Force, Momentum & Impulse

Conservation of Linear Momentum

Case I: Collisions with separation after the collision.

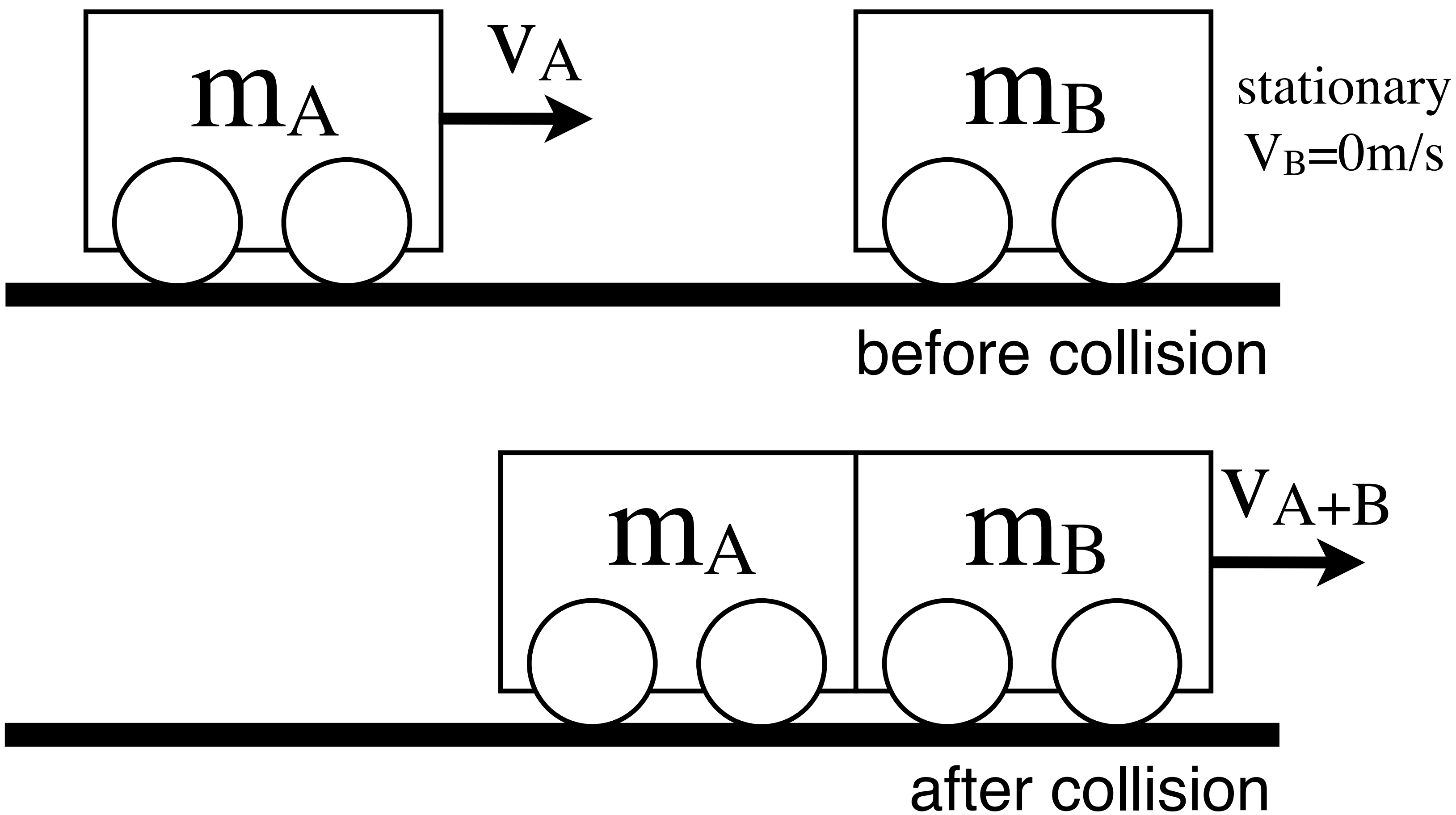


$$m_A v_{iA} + m_B v_{iB} = m_A v_{fA} + m_B v_{fB}$$

Force, Momentum & Impulse

Conservation of Linear Momentum

Case 2: Collisions with coupling (i.e. moving together after collision)

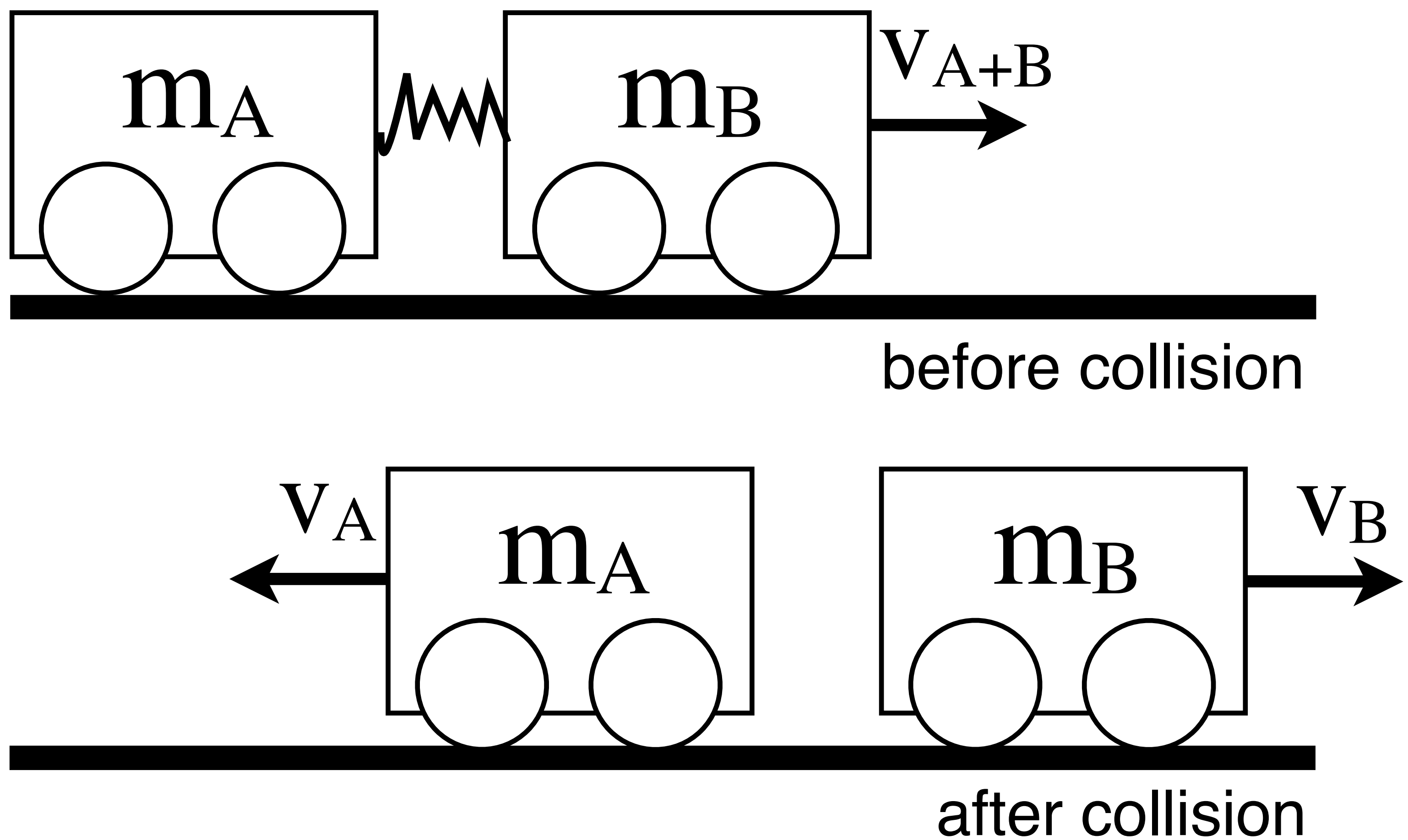


$$m_A v_{iA} + m_B v_{iB} = (m_A + m_B) v_f$$

Force, Momentum & Impulse

Conservation of Linear Momentum

Case 3: Explosions (i.e. move together before and then separate)



$$(m_A + m_B)v_i = m_A v_{fA} + m_B v_{fB}$$

Force, Momentum & Impulse

Some other things you should remember from grade 11

Newton's second law

Some new things to observe

Impulse is defined as the change in momentum of a system. It is not considered a mathematical quantity but rather a natural one.

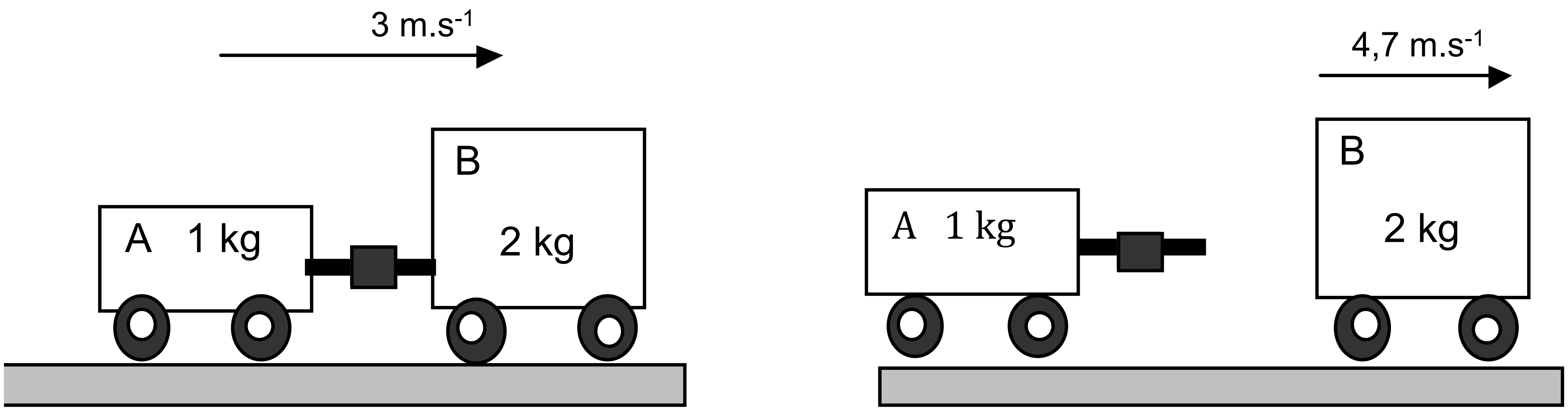
An external resultant force causes an object to undergo a change in momentum. This relationship of impulse is useful in explanation some of our everyday applications that are used in various apparatus to reduce injury and risk.

$$F_{net}\Delta t = \Delta p = m(v_f - v_i)$$

Conservation of Linear Momentum - Practice Question 1

A combination of trolley A (fitted with a spring) of mass 1 kg, and trolley B of mass 2 kg, move to the right at 3 $m.s^{-1}$ along a frictionless horizontal surface. The spring is kept compressed between the two trolleys.

While the combination of the two trolleys is moving at 3 $m.s^{-1}$, the spring is released and when it has expanded completely, the 2 kg trolley is then moving to the right at 4,7 $m.s^{-1}$ as shown below.



(a) State the principle used to calculate the velocity of the 1kg trolley immediately after the spring has expanded completely.

(c) Calculate the magnitude and direction of the velocity of the 1kg trolley immediately after the spring has expanded completely.

Conservation of Linear Momentum - Practice Question 2

A toy train, mass 0,5 kg is moving along a toy track to the right with a speed of $2 m.s^{-1}$. It collides with another stationary train that also has a mass of 0,5 kg. As a result of the collision, the first train remains stationary.

(a) Use the principle of conservation of momentum to find the velocity of the second train after the collision.

(b) Calculate the total kinetic energy before and after the collision. How do they compare with each other

Conservation of Linear Momentum - Practice Question 3

A bullet of mass 50g travelling horizontally at $500 m.s^{-1}$ strikes a stationary wooden block of mass 2kg resting on a smooth horizontal surface. The bullet goes through the block and comes out on the other side at $200 m.s^{-1}$. Calculate the speed of the block after the bullet has come out the other side.

Conservation of Linear Momentum - Practice Question 4

A bullet of mass 50 g travelling horizontally at 600m.s^{-1} strikes a stationary wooden block of mass 2 kg resting on a smooth horizontal surface. The bullet gets stuck in the block.

(a) Name and state the principle which can be applied to find the speed of the block-and-bullet system after the bullet entered the block.

(b) Calculate the speed of the bullet-and-block system immediately after impact.

(c) If the time of impact was 5x10^{-4} seconds, calculate the force that the bullet exerts on the block during impact.

see handout for more practice questions

Vertical Projectile Motion

Vertical Projectile Motion

an introduction

Projectile: an object propelled through the air

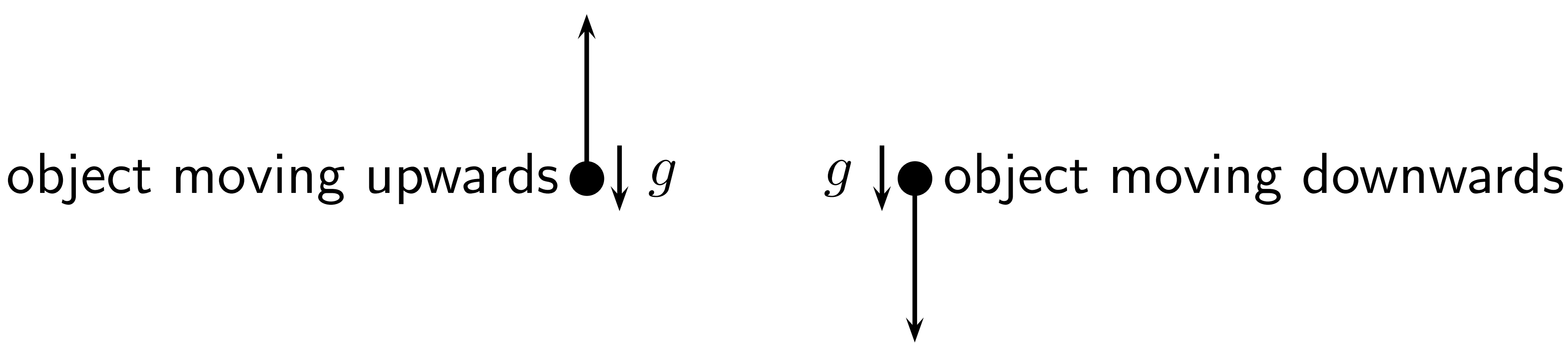
We will consider objects in a gravitational field **near the earth's surface** that is **thrown straight up and falls back down** (i.e. objects that are displaced vertically only).

We will ignore the effects of air friction.

Vertical Projectile Motion

an introduction

When an object is in a gravitational field, it ***always accelerates downwards*** with a constant acceleration g whether the object is moving upward or downward.



This means that if an object is moving upwards, it decreases until it stops ($v_f = 0$ m·s^{-1}). This is the maximum height that the object reaches, because after this, the object starts to fall.

Important: Projectiles have zero velocity at their greatest height.

Important: Projectiles take the same the time to reach their greatest height from the point of upward launch as the time they take to fall back to the point of launch.

Vertical Projectile Motion

an introduction

We will consider three cases of vertical motion. For our calculations we will use upward as positive and downward motion as negative. However, you may choose which ever system you feel comfortable with as long as you state this at the beginning of you calculation or unless the questions states otherwise.

$v_i = 0$ m/s

$g = -9.8$ m/s^2

$\Delta x = ?$

$\Delta t = ?$

$v_f = ?$

(Case 1)

$v_f = 0$ m/s	$v_i = 0$ m/s
$g = -9.8$ m/s^2	$g = -9.8$ m/s^2
$\Delta x = ?$ (+'ve)	$\Delta x = ?$ (-'ve)
$\Delta t = ?$ (+'ve)	$\Delta t = ?$ (+'ve)
$v_i = ?$ (+'ve)	$v_f = ?$ (-'ve)

(Case 2)

Vertical Projectile Motion

an introduction

(Case 3 - by separating the motion)

$v_f = 0$ m/s

$g = -9.8$ m/s^2

$\Delta x = ?$ (+'ve)

$\Delta t = ?$ (+'ve)

$v_i = ?$ (+'ve)

$v_i = 0$ m/s

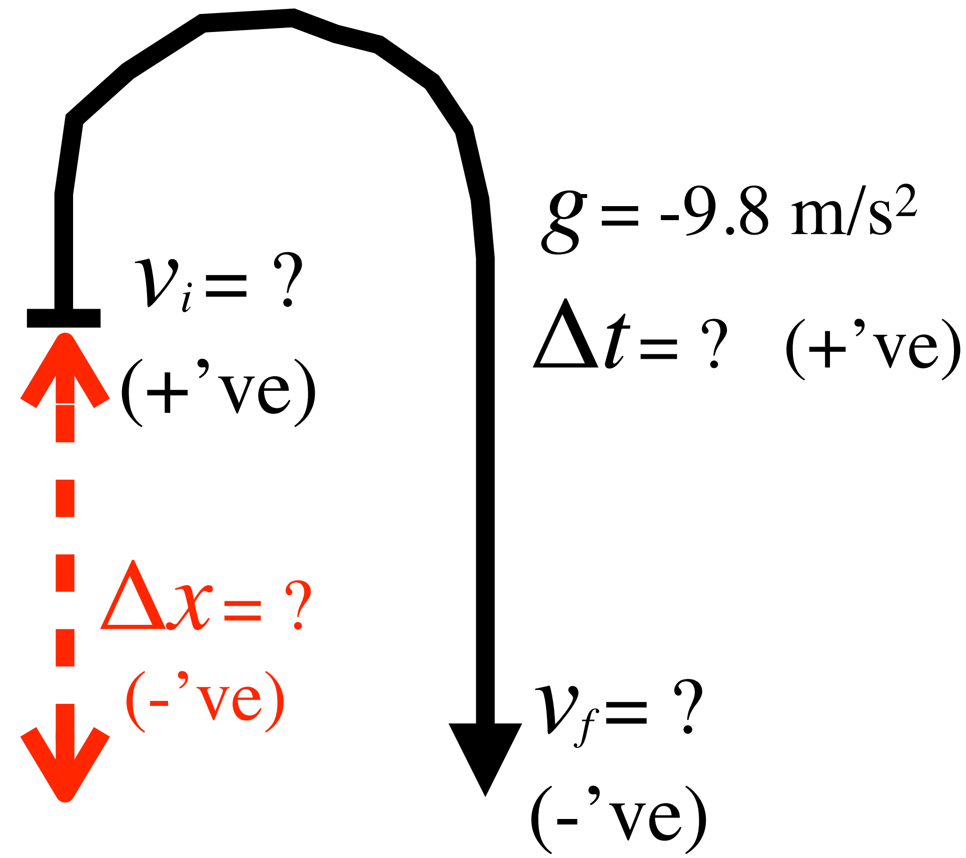
$g = -9.8$ m/s^2

$\Delta x = ?$ (-'ve)

$\Delta t = ?$ (+'ve)

$v_f = ?$ (-'ve)

(Case 3 - by combined motion calculations)



Vertical Projectile Motion

the equations of uniformly accelerated linear motion

In solving VPM problems we may use the equations of uniformly accelerated linear motion. These are the same formulae used for linear motion and free fall that you studied in grades 10 and 11.

$$\begin{aligned} v_f &= v_i + gt \\ \Delta x &= \frac{(v_i + v_f)}{2} t \\ \Delta x &= v_i t + \frac{1}{2} g t^2 \\ v_f^2 &= v_i^2 + 2g\Delta x \end{aligned}$$

$$\begin{aligned} v_i &= \text{initial velocity (m·s}^{-1}\text{) at } t = 0 \text{ s} \\ v_f &= \text{final velocity (m·s}^{-1}\text{) at time } t \\ \Delta x &= \text{height above ground (m)} \\ t &= \text{time (s)} \\ \Delta t &= \text{time interval (s)} \\ g &= \text{acceleration due to gravity (m·s}^{-2}\text{)} \end{aligned}$$

intro practice questions

$$v_f = v_i + gt$$
$$\Delta x = \frac{(v_i + v_f)}{2}t$$
$$\Delta x = v_i t + \frac{1}{2}gt^2$$
$$v_f^2 = v_i^2 + 2g\Delta x$$

Question 1: A ball is thrown upwards with an initial velocity of 10 m.s^{-1}.

(a) Determine the maximum height reached above the thrower's hand.

(b) Determine the time it takes the ball to reach its maximum height.

Question 2: A cricketer hits a cricket ball from the ground so that it goes directly upwards. If the ball takes, 10 seconds to return to the ground, determine its maximum height.

BTG

intro practice questions

$$v_f = v_i + gt$$

$$\Delta x = \frac{(v_i + v_f)}{2}t$$

$$\Delta x = v_i t + \frac{1}{2}gt^2$$

$$v_f^2 = v_i^2 + 2g\Delta x$$

Question 3: A cricketer hits a cricket ball straight up into the air. The cricket ball has an initial velocity of 20 $m.s^{-1}$.

(a) What height does the ball reach before it stops to fall back to the ground.

(b) How long has the ball been in the air for?

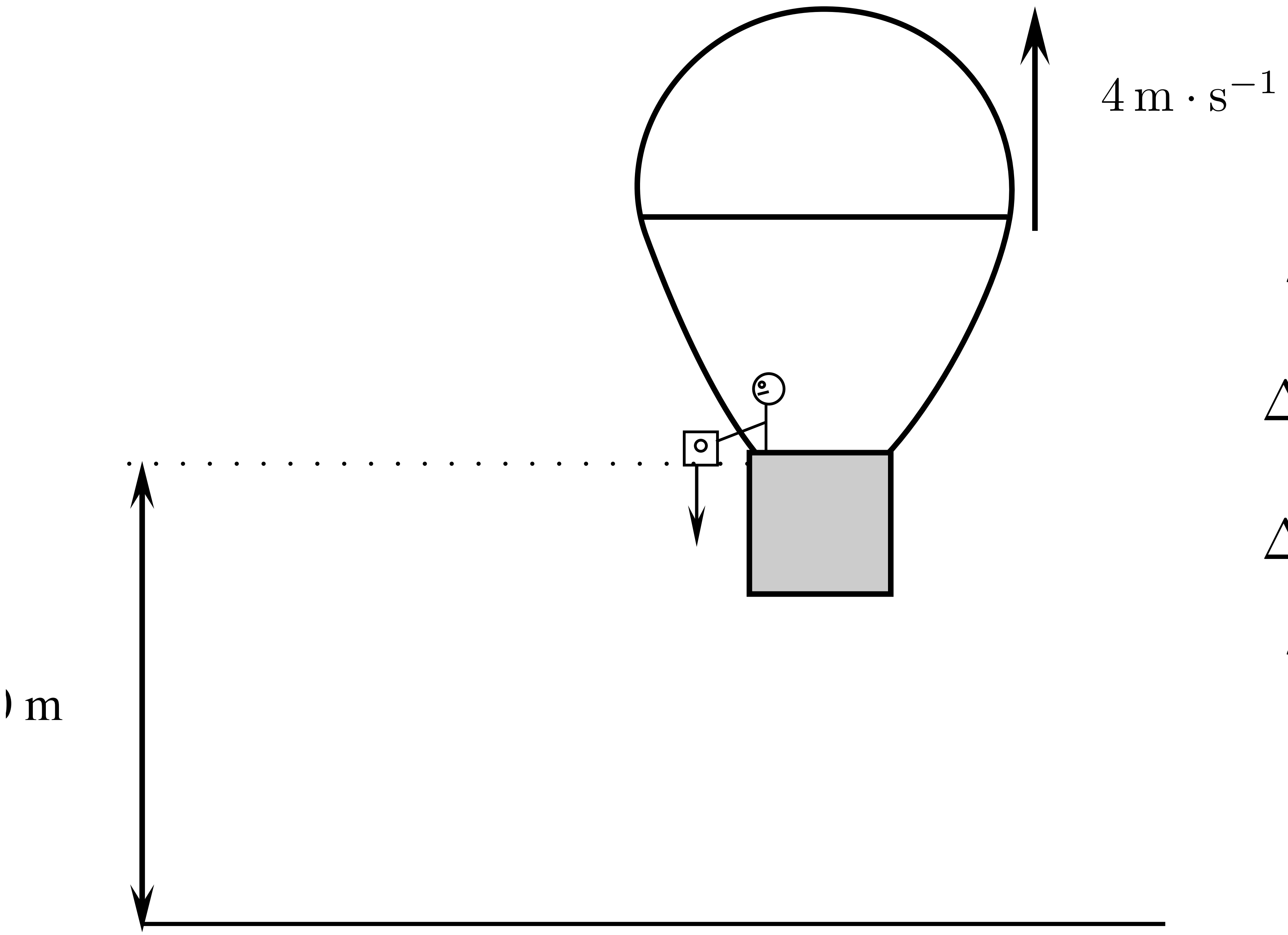
Question 4: Zingi throws a tennis ball up into the air. It reaches a height of 80 cm.

(a) Determine the initial velocity of the tennis ball.

(b) How long does the ball take to reach its maximum height?

intro practice questions **Question 5**

A tourist takes a trip in a hot air balloon. The hot air balloon is ascending (moving up) at a velocity of 4 m·s^{-1}. He accidentally drops his camera over the side of the balloon's basket, at a height of 20 m. Calculate the velocity with which the camera hits the ground.



$$v_f = v_i + gt$$

$$\Delta x = \frac{(v_i + v_f)}{2}t$$

$$\Delta x = v_i t + \frac{1}{2}gt^2$$

$$v_f^2 = v_i^2 + 2g\Delta x$$

Vertical Projectile Motion

graphs of vertical projectile motion

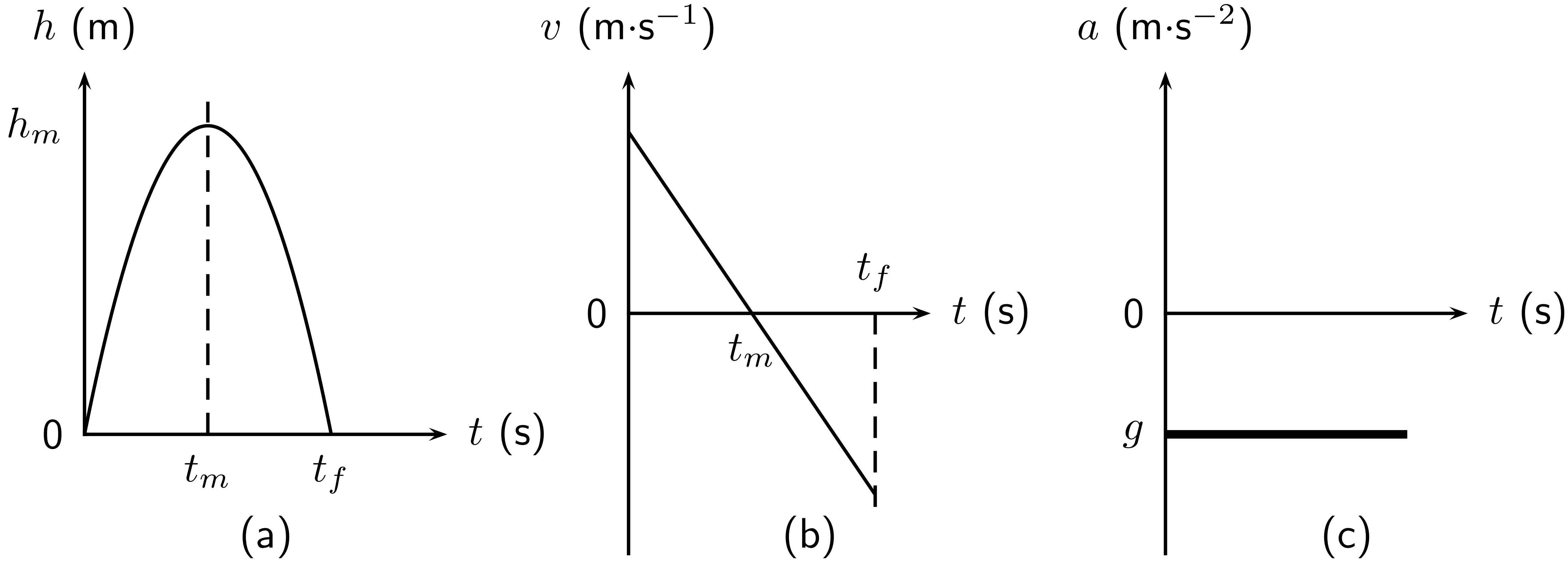


Figure 21.3: Graphs for an object thrown upwards with an initial velocity v_i. The object takes t_m s to reach its maximum height of h_m m after which it falls back to the ground. (a) position vs. time graph (b) velocity vs. time graph (c) acceleration vs. time graph.

Graphs of VPM practice questions

$$
\begin{aligned}
v_f &= v_i + gt \\
\Delta x &= \frac{(v_i + v_f)}{2}t \\
\Delta x &= v_i t + \frac{1}{2}gt^2 \\
v_f^2 &= v_i^2 + 2g\Delta x
\end{aligned}
$$

Question 1: Question: Stanley is standing on the a balcony 20 m above the ground. Stanley tosses up a rubber ball with an initial velocity of 4,9 m.s^{-1}. The ball travels upwards and then falls to the ground. Draw graphs of position vs. time, velocity vs. time and acceleration vs. time. Choose upwards as the positive direction.

Graphs of VPM practice questions

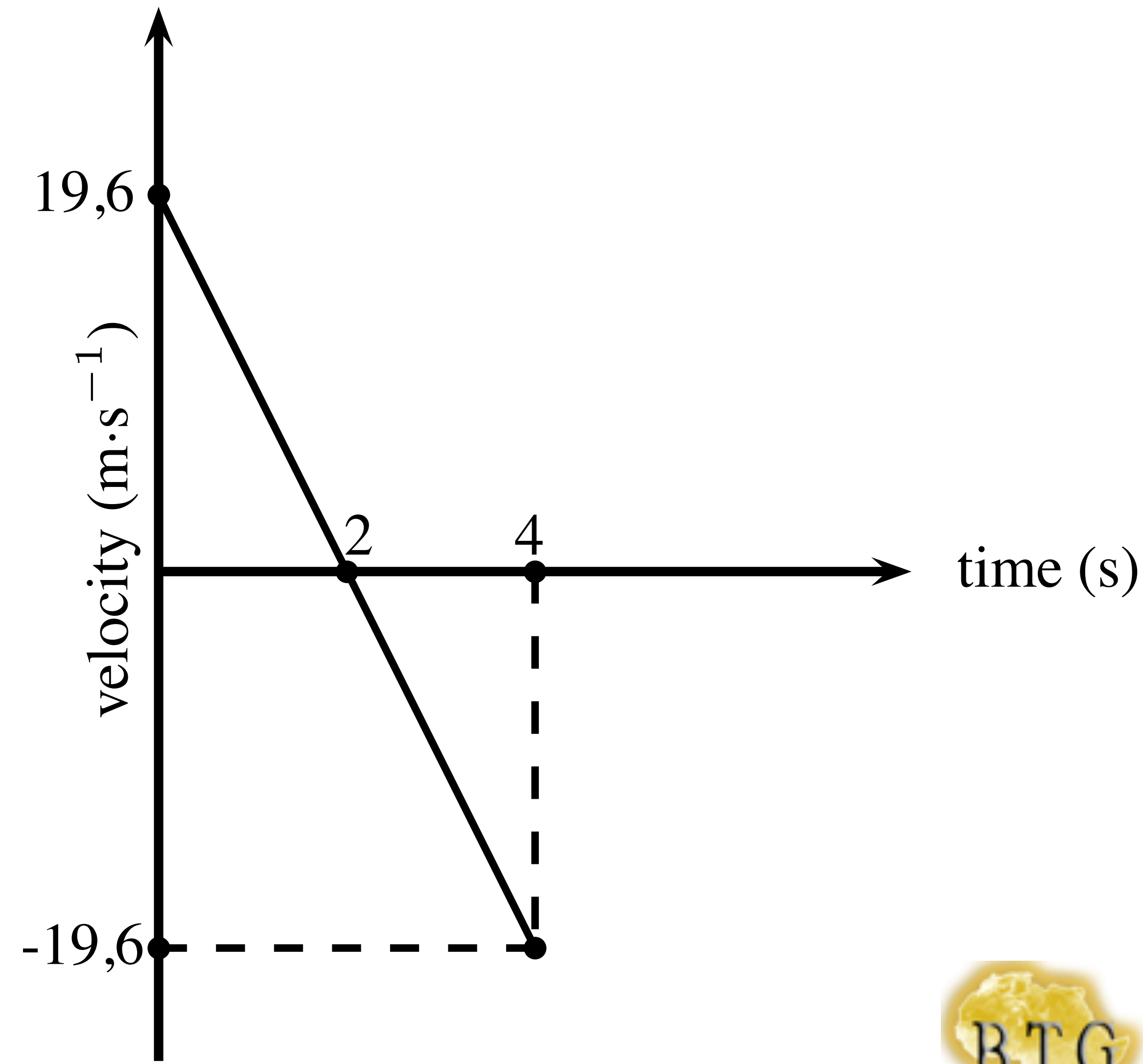
$$v_f = v_i + gt$$
$$\Delta x = \frac{(v_i + v_f)}{2}t$$
$$\Delta x = v_i t + \frac{1}{2}gt^2$$
$$v_f^2 = v_i^2 + 2g\Delta x$$

Question 2: Question: A cricketer hits a cricket ball from the ground and the following graph of velocity vs. time was drawn. Upwards was taken as positive. Study the graph and answer the following questions:

(a) Describe the motion of the ball according to the graph.

(b) Draw a sketch graph of the corresponding displacement-time graph. Label the axes.

(c) Draw a sketch graph of the corresponding acceleration-time graph. Label the axes.



Graphs of VPM practice questions

$$v_f = v_i + gt$$

$$\Delta x = \frac{(v_i + v_f)}{2}t$$

$$\Delta x = v_i t + \frac{1}{2}gt^2$$

$$v_f^2 = v_i^2 + 2g\Delta x$$

Question 3: The graph below (not drawn to scale) shows the motion of tennis ball that was thrown vertically upwards from an open window some distance from the ground. It takes the ball 0,2 s to reach its highest point before falling back to the ground. Study the graph given and calculate

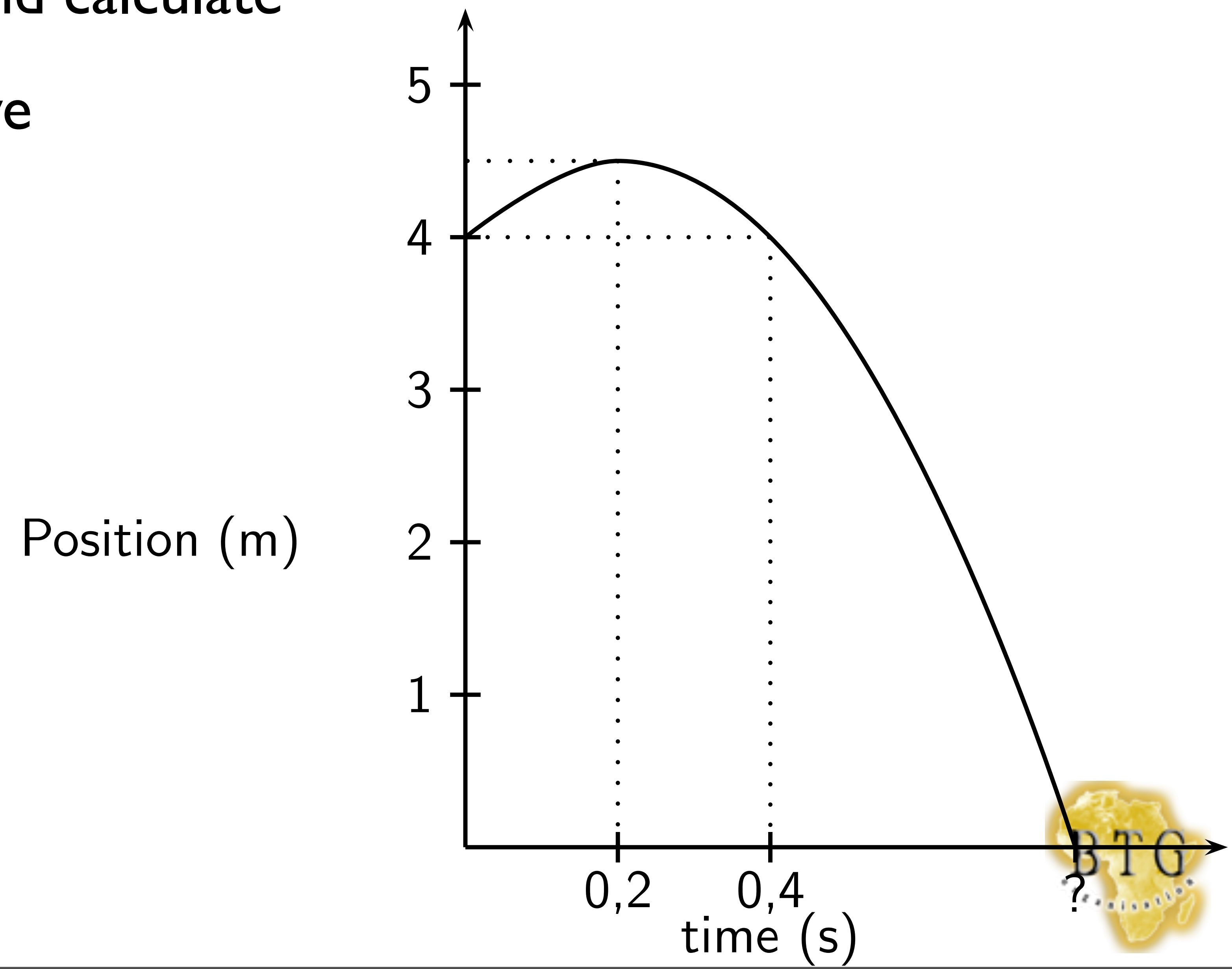
(a) how high the window is above the ground.

(b) the time it takes the ball to reach the maximum height.

(c) the initial velocity of the ball.

(d) the maximum height that the ball reaches.

(e) the final velocity of the ball when it reaches the ground.



Graphs of VPM practice questions

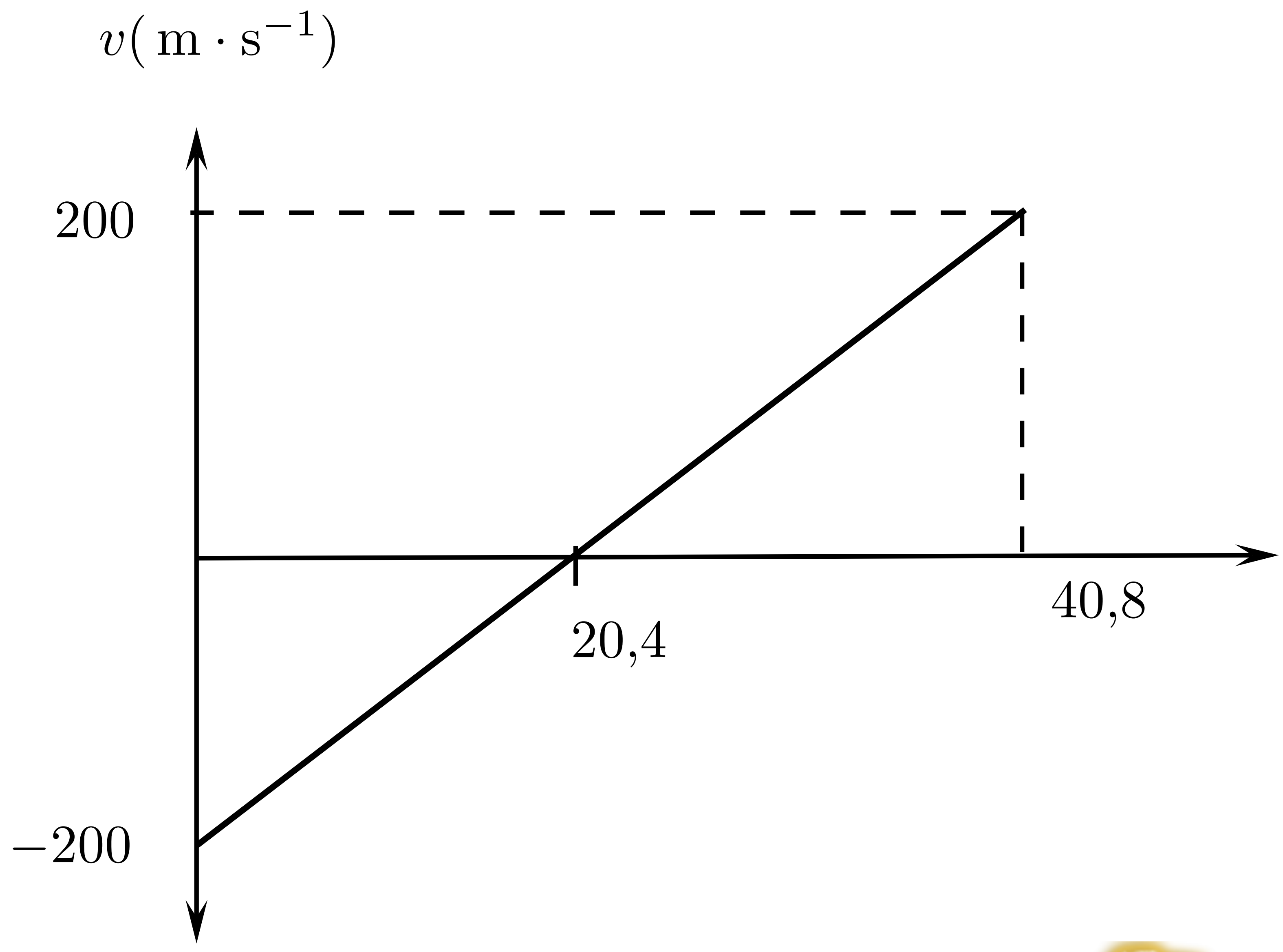
$$v_f = v_i + gt$$
$$\Delta x = \frac{(v_i + v_f)}{2}t$$
$$\Delta x = v_i t + \frac{1}{2}gt^2$$
$$v_f^2 = v_i^2 + 2g\Delta x$$

Question 4: A bullet is shot from a gun. The following graph is drawn. Downwards was chosen as positive

(a) Describe the motion of the bullet

(c) Draw a displacement - time graph

(e) Draw an acceleration - time graph



Graphs of VPM practice questions

$$v_f = v_i + gt$$

$$\Delta x = \frac{(v_i + v_f)}{2}t$$

$$\Delta x = v_i t + \frac{1}{2}gt^2$$

$$v_f^2 = v_i^2 + 2g\Delta x$$

Question 5: Amanda throws a tennis ball from a height of 1,5m straight up into the air and then lets it fall to the ground.

Draw graphs of

Δx vs t ; *v* vs t and *a* vs t

for the motion of the ball. The initial velocity of the tennis ball is 2 m.s^{-1}. Choose upwards as positive.